Asia Pacific Education Review
2004, Vol. 5, No. 2, 157-166.

Pre-Service Teachers' Mathematical Reasoning as an Imperative for Codified Conceptual Pedagogy in Algebra: A case Study in Teacher Education

Belinda de Castro
University of Santo Tomas
Philippines

Through the use of taped interviews, the reasoning level of eleven (11) pre-service teachers relative to selected concepts in Algebra was ascertained. Yumus' (2001) levels of reasoning were applied as a guide, namely: (a) Level 1: Unable to produce any reasoning, (b) Level 2: Have awareness of the models, known facts, properties and relationships to be used but cannot produce any arguments; (c) Level 3: Able to produce some reasoning although the arguments are weak and (d) Level 4: Able to produce strong arguments to support their reasoning. Using this guide it was found that of the 121 responses given, 47.1% were at level 1, 29.8% at level 2, 16.5% at level 3 and only 6.6% were at level 4. The most difficult problem proved to be converting repeating decimals to fractions, while the easiest was on finding the value of x^0. As a whole, the reasoning ability of the respondents, based to their average reasoning ability on the given tasks, indicate that 73% was low, 27% was moderate and that nobody had a high level of reasoning. Assessments followed as to the factors contributing to this situation and possible solutions.

Key Words: mathematical reasoning, teaching and learning mathematics, pre-service school teachers levels of reasoning in mathematics

The vision for mathematics education, as described in the *Principles and Standards for School Mathematics* (NCTM Standards, 2000), is very ambitious. It describes solid mathematics curricula, with competent and knowledgeable teachers and adequate and advanced resources which offer opportunities for students to learn mathematical concepts and procedures in-depth and develop higher order thinking skills. It emphasizes the use of reasoning to enable students to go beyond mere memorization of facts, rules and procedures (NCTM Standards, 2000). This is one distinguishing feature of mathematics that makes use of structural organization by which the parts of mathematics are connected to each other, and not just to the real world objects of our experience (Raimi, 2002).

The success of the current reform movement in mathematics education as prescribed by the NCTM Standards (2000) depends on teachers' conceptual knowledge of the school mathematics subject matter. The development of a deep understanding of the school mathematics curriculum and how it fits within the discipline is central to teachers' pedagogical preparation. Oftentimes, it is tacitly assumed that teachers' knowledge of the content of school mathematics is enough by the time they complete their own learning experience. Teachers need opportunities to revisit their school mathematics topics in ways that will allow them to develop a deeper understanding of school mathematics subject matter required for teaching that subject matter (Bryan, n.d.). They also need to understand their students' thought processes so that they can develop conceptions of the typical trajectories of

Belinda V. de Castro, Faculty Researcher at the Center for Educational Research and Development and the College of Commerce and Accountancy, University of Santo Tomas. Correspondence concerning this article should be addressed to Center for Educational Research and Development, University of Santo Tomas, Espana, Manila, Philippines 1008. Electronic mail may be sent to bheldc@ yahoo.com

students' learning and can use this knowledge as landmarks of understanding in individuals (Murray, n.d.).

The teacher's goal is to provide meaningful learning for the students, and the student's learning objective is to be proficient in mathematics. This is an ideal scenario in the teaching-learning environment. When students understand mathematics, they can use their knowledge flexibly. They are able to combine factual knowledge, procedural facility and conceptual understanding in meaningful ways. Learning the 'basics' is important. Students who memorize facts or procedures without understanding are usually not sure when or how to use what they know. In contrast, meaningful learning enables students to handle novel problems and settings. Learning with understanding helps students become autonomous learners, because they learn more and do so more effectively when they take control of their own learning (Ernest, 1994; Kauchak and Eggen, 1998).

It is alarming to note that among the 38 countries who participated in the Third International Mathematics and Science Study – Repeat (TIMSS-R), the Philippines was ranked among the lowest three. The said study covered five content areas, namely, Fractions and Number Series, Measurement, Data Representation, Analysis and Probability, Algebra and Geometry. The lowest performance of the Philippines was in Algebra, where the mean was 349, compared to the international average of 489 (Ibe, 2001; TIMSS-R, 2000). Ibe (2001) attributed this poor performance of Filipino students to the teachers' methods of imparting knowledge, namely, their questioning techniques, their stance and approach, their development and illustration of concepts and problem solving and textbook treatment of the concepts. Teachers emphasize algorithmic solutions instead of utilizing students' meaning, visualization, and interpretation of given word problems. Additionally, they seldom ask higher-order questions which can be used to train students to apply information and principles, rather than simply recall facts or isolated rules (Greenes, 1995; Murray, n.d.).

The foregoing observations invite researchers and classroom teachers to take a closer look at how pedagogical content knowledge is incarnated by pre-service teachers in teaching certain Algebra concepts, where the country had the lowest performance in the TIMSS-R.

Review of Related Literature

To succeed in higher mathematics, students should proceed from mere memorization and drill to abstract mathematical reasoning. Math curricula focus less on computation and more on mathematical reasoning (NCTM Standards, 2000). It is a mental habit, and like all habits, it must be developed through consistent use in many contexts over a number of years (Baroody, 1993; Fraivillig, 1999; Murray, n.d.; Raimi, 2002).

Shulman, as cited by Orton (1997), described a cyclical model of sound reasoning that includes the phases of comprehending the subject matter, transforming it into a form that students can understand, delivering the instruction, evaluating the performance and then reflecting on the teaching, which leads to a new phase of comprehension. Sound reasoning requires both a process of thinking about what and an adequate base of facts, principles and experiences from which to reason.

O'Daffler and Thornquist, as cited by Yumus (2001), defined mathematical reasoning as a part of mathematical thinking that involves forming generalizations and drawing valid conclusions about ideas and how they are related. Hyde, in the same study of Yumus (2001), highlighted that reasoning is one of the key areas in mathematical thinking, apart from communication, metcognition and problem solving, which involves making sense in specific situations and enforcing meaning in relation to existing schemata. People who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects. They ask if those patterns are accidental or if they occur for a particular reason; they hypothesize, seek supporting evidence for their ideas and finally draw reasoned conclusions (Bigge & Shermis, 1999). Reasoning is the foundation of mathematics (Baroody, 1993; Krulick & Rudnick, 1996; Raimi, 2002; Siegel, 1988). While scientific phenomena are verified through observation, mathematics relies on logic. (Steen, 1999).

Baroody, Wilson, Kauchak and Eggen, as cited by de Castro (2003), state that teaching through reasoning, just like the reasoning used proving theorems in geometry, is of two types namely: inductive and deductive reasoning. Inductive reasoning involves perceiving regularity in the samples given. Finding a commonality among diverse samples is a basis for concept formation. Deductive reasoning is simply a matter of drawing conclusions that necessarily follow from presently known information. Thus, teaching through mathematical reasoning refers to the proof, the basis and the explanations that teachers use to build mathematical concepts, rules or principles in the minds of students. Students can retain learned knowledge longer and apply it to related problems when they really understand, and are not merely instructed to accept and follow the algorithm (Bromme & Steinbring, 1994; Vacc & Bright, 1999).

Malaysia performed above the international standard in the TIMSS-R (16^{th} place out of 38 countries). Their students performed well on questions that require algorithmic facility but not on questions that require application of concepts, word problems and problems involving diagrams. The need to look into the teachers’ stance of imparting knowledge as to the reason behind the students’ lack of understanding of some concepts prompted the generation of the four levels of reasoning by Yumus (2001) in her study on mathematical reasoning. These are: (a) Level 1: Unable to produce any reasoning, (b) Level 2: Have awareness of the models, known facts, properties and relationships to be used but cannot produce any arguments; (c) Level 3: Able to produce some reasoning although the arguments are weak and (d) Level 4: Able to produce strong arguments to support their reasoning. It was found out that the ability of pre-service teachers in giving reasons to school mathematics tasks is generally low. It appears that the four years spent learning university-level mathematics did not assist most of them in developing insights into tasks that they had encountered in school mathematics.

Bryan’s study (n.d.) also explored and described pre-service secondary mathematics teachers’ knowledge of school mathematics and found that it was generally lacking in conceptual depth. He subjected his respondents to paper and pencil computations and clinical interviews and categorized their responses as SPP, showed procedural proficiency; CCB, considered conceptual basis; ONE, offered no explanation; OFE, offered flawed explanation; and OSE, offered sound explanation.

Doerr (2003) made use of a detailed analysis of the mathematical reasoning development and a macrolevel analysis of the diversity of thinking patterns of a small group of students across a sequence of tasks to devise a modeling approach for the teaching and learning of mathematics. This shifted the focus of the learning activity from finding a solution to a particular problem to creating a system of relationships that can be used to generalize and are reusable. Students reasoning on the relationships between and among quantities and their applicability in related situations are taken into consideration. Results suggest that students were able to create generalizable and reusable models on certain concepts like selecting, ranking and weighting data with the use of mathematical reasoning.

Ball (2000), for his part, posited that explanation works only if it is at a sufficient level of granularity, and if it includes the necessary steps for reasoning to make sense for a particular learner or the whole class, based on what they currently know and do not know. Explicit connections between the mathematical ideas and activities with their prior knowledge play an important role in engaging students in high-level thought processes (Entwistle, 1998; Greenes, 1995; Hemingson & Stein, 1997; Jones, Wilson, & Bhojwani, 1997).

As to pedagogy, Siegel (1988) stated that a teacher who utilizes the critical manner of teaching encourages in his students the skills, habits and disposition essential to the development of the critical spirit. This means that the teacher should always respect the right of the student to question and demand reasons, and consequently recognize the obligation to provide reasons whenever demanded. This expresses the teacher’s a willingness to subject all beliefs and practices to scrutiny and allow students genuine opportunity to understand the role which reason plays in the justification of thought and action. With this, the teacher needs to be well grounded on the school mathematics that he will teach to his students.

The Present Study

Pre-service preparation is the foundation of mathematics teaching, but it gives teachers only a small amount of what they need to know and understand throughout their career. Finding ways of integrating knowledge and practice is essential if teachers are to develop the resources they need for their work (Fairbanks, Freedman, & Kahn, 2000; Mewborn, 1999). A number of completed studies on the knowledge of prospective secondary mathematics teachers have shown similar results. These results reveal that even with a completed substantial amount of university mathematics coursework, these prospective teachers may still lack the level of conceptual understanding that is required for the teaching of that subject matter, in ways consistent with those advocated by the current reform movement in mathematics (Bryan, n.d.).

The purpose of this study is to take a closer look into the preparedness of our own pre-service teachers in teaching certain Algebra concepts. A lack in pedagogical content knowledge in delivering content due to underexposure in the teaching of school mathematics was taken into consideration during the investigation.

The following research questions served as a guide in the gathering and analysis of data: (1) What is the level of reasoning that each respondent exhibited in each mathematical task?; (2) What is the overall level of reasoning evidenced by each respondent when interpreting and justifying the mathematical tasks?; (3) What types of tasks are most difficult for the pre-service teachers to apply reasoning to?; and (4) What measures can better upgrade the teaching of mathematical tasks, thus facilitating students’ learning and understanding?

Study Framework

This study aims to stress the importance of mathematical reasoning in order to shift prospective teachers' instructional explanations from instrumental understanding, which refers to the "what and how" of mathematics (the rules without reasons), to relational understanding of mathematics, which includes insights to the "why's" of mathematics (the reason for the what and how). The relevance of the use of reasoning in the teaching learning process and the various levels of reasoning teachers and students undergo to achieve meaningful learning are encapsulated in the paradigm below:

As the paradigm indicates, teachers bring with them their content and pedagogical knowledge to assist them in the knowledge transmission process. They manifest diverse levels of reasoning as they explain the subject matter to their students. Teachers, making use of the lowest level of reasoning for their teaching, may only exhibit procedural facility, that is making use of rules without reasons. On the other hand, the students come in with their prior knowledge and varied levels of understanding. Students' reasoning may not be clearly evident during their first encounter with the concept. As teachers make the concepts clearer through the use of mathematical reasoning, that is, by making connections to new ideas, the concepts become clearer. However, we can only say that teachers have been effective in their teaching and students have achieved meaningful learning if they have reached the final level of reasoning, that is, they were able to produce strong arguments that support the reasons as to the "why's and how's" of the concept.

To go beyond rote memorization in order to achieve depth of understanding, students need to develop and integrate a network of associations linking new input to preexisting knowledge and beliefs. Hence, teaching involves inducing conceptual change in students, not infusing knowledge into a vacuum. The aim of teaching with mathematical reasoning is for developing students' mastery of content, to understand the logic of the content, and, most significantly, to apply knowledge in new and unfamiliar circumstances (Cooney, 1998; Doerr, 2003; Education Issues Series, 1996; Kinach, 2002; Vacc & Bright, 1999). Thus, their twin goals of effective teaching and meaningful learning are achieved.

Methodology

Participants

Eleven (11) students undertaking practicum at a reputable

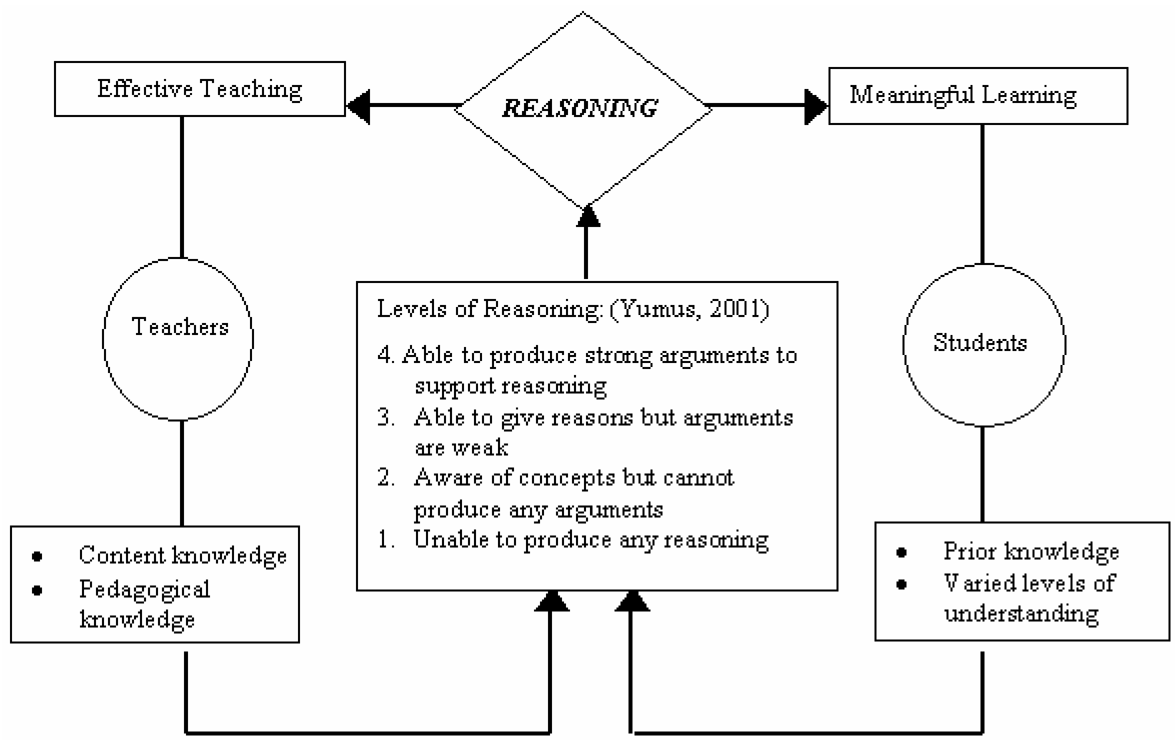

Figure 1. Mathematical Reasoning: Imperative Factor in the Teaching-Learning Process

teacher education institution were randomly chosen as respondents. The group represents about 30% of the graduating class in the said institution, majoring in mathematics education. The pre-service teachers were considered the most qualified respondents for the study due to the fact that they took their academic courses only recently, and are already practicing their teaching skills.

Instrumentation

A set of eleven (11) algebraic concepts, normally left unexplained by teachers and "to be accepted as fact", was considered for interview purposes. The conceptual understanding of the respondents for the mathematical ideas listed below was the focal point of the exploration conducted during the interviews.

Table 1. *Interview questions*

Task	Interview questions
1	Solve 4/6 x 3/4. Explain the algorithm.
2	Solve 6/15 ÷ 1/5 . Explain the algorithm.
3	Show why – (-x) = x
4	Show why $x^0 = 1$ (Anything raised to the zero power is equal to 1)
5	Solve and explain each of the following: a. 0/2 b. 2/0 c. 0/0
6	Convert $4.\overline{1515}\ldots$ to a fraction. Explain the algorithm.
7	Given that $(2.6)^2 = 6.76$, which of the following can be solved without using the calculator or a table? a. $\sqrt{6.76}$ b. $\sqrt{67.6}$ c. $\sqrt{676}$ d. $\sqrt{6760}$ e. $\sqrt{67600}$
8	Find the sum of 10 terms in the arithmetic progression 12, 9, 6, 3,… Explain the algorithm.
9	Find the sum of 10 terms in the geometric sequence 2, 6, 18,… Explain the algorithm.
10	Explain why $\log_{10} XY = \log_{10}X + \log_{10}Y$
11	Solve 3 + 4 ÷ 2 x 2 – 1 Explain why the algorithm works.

Procedure

Data from this qualitative study were obtained through clinical interviews. Student teachers were interviewed on a one-to-one basis within a 1 to 2 hour session. They were asked to explain the 11 mathematical tasks and give reasons for the algorithm used. Before the respondents were interviewed, they were first asked to solve problems related to the tasks. They were allowed to refer to the textbook to help them recall relationships and properties they will need to provide explanations for in the interview. Data consisted of interview transcripts, the respondents' written responses on the answer sheets, and the interviewer's notes on the respondents non-verbal behavior. All interview sessions were audio-taped for transcription purposes. The adequacy of the prospective teachers' instructional explanations and the depth of their understanding of algebraic concepts in the enumerated tasks were analyzed by three teacher experts, including the researcher, using the following scale adapted from the study of Yumus (2001).

Levels of reasoning considered were that:

1 : Unable to produce any reasoning.

2 : Aware of models, known facts, properties and relationships used as basis of reasoning, but cannot produce any arguments.

3 : Able to provide reasons although arguments are weak.

4 : Able to provide strong arguments to support reasoning.

Results and Discussion

Table 3 shows the level of reasoning of each respondent on each of the given mathematical tasks.

The Table indicates that out of the 121 responses from the 11 student respondents, it was Task 6 on converting repeating decimals to fractions which they found most difficult to deal with and Task 4 on finding the value of x^0, which they found easiest to reason out. Out of those 121 responses, the respondents had basically "no explanation" for the algebraic concept being considered 57 times (level 1–47.1%); comments seemed more appropriately characterized as superficial rather than conceptual 36 times (level 2–29.8%); offered an explanation that was "flawed" in one or more ways 20 times (level 3–16.5%) and successfully offered an explanation categorized as "conceptually sound" only 8 times (level 4 – 6.6%). This shows that majority of these respondents could not give any reasons for the said mathematical tasks.

Table 3. *Respondents' Level of Reasoning across Tasks*

	Task											
Student	1	2	3	4	5	6	7	8	9	10	11	Mean
1	2	2	2	4	3	1	1	1	1	2	1	1.82
2	2	3	3	4	3	1	1	1	1	1	1	1.91
3	2	2	2	4	2	1	1	2	1	1	1	1.73
4	3	3	3	4	3	1	2	1	1	1	1	2.09
5	2	2	2	4	3	1	1	3	2	1	1	2.00
6	2	2	1	1	1	1	1	1	1	1	2	1.27
7	3	2	2	4	1	1	1	1	1	1	2	1.73
8	3	2	2	4	3	1	1	2	2	1	2	2.09
9	3	2	2	1	3	1	1	2	2	1	3	1.91
10	3	2	1	4	2	1	1	1	1	1	1	1.64
11	2	2	3	1	3	1	3	2	2	1	1	1.91
Mean	2.4	2.1	2.09	3.1	2.4	1.0	1.27	1.5	1.3	1.0	1.4	1.83

Table 4: *Respondents' Reasoning Ability*

Average Reasoning Ability	Verbal Interpretation	n (%)
1 – 1.99	Low	8 (73%)
2 – 2.99	Moderate	3 (27%)
3 – 4.0	High	0 (0%)

Table 4 shows the reasoning ability of the respondents when grouped according to their average reasoning ability on the given tasks, indicating that 73% of them have a low reasoning level, 27% have a moderate reasoning level and that none of the subjects exhibited a high level of reasoning. This tallies with the results of the studies made by Yumus (2001) and Bryan (n.d.), where pre-service teachers also demonstrated a low reasoning level when exposed to some mathematical tasks.

Certainly, a summary of how responses were categorized as in the previous tables serves only to provide a rough glimpse of the wealth of information about these eleven (11) pre-service teachers' conceptual knowledge that was revealed in the extensive dialogues contained in the interview transcriptions. To illuminate some of the more general findings in this study, here are some excerpts from the interviews with the student respondents. ("I" refers to interviewee, "R" refers to the respondent)

On Multiplication and Division of Fractions. Most of the student respondents were able to perform the task and discuss the step by step procedure for the algorithm but could not give any reason as to how and why the algorithm works. Only 5 (45%) for multiplication and 2 (18%) out of 11 for division were able to give reasons but their arguments are weak. Here are some transcriptions of the interview.

I: How will you explain division of fractions to your students?

R: First we have to get the reciprocal of the second fraction. What I mean is to change the operation symbol to multiplication.
I: What is the reason behind it? Why do you think it works?
R: Because if we simply divide the two fractions, it will be difficult. (thinks) How do I explain that?
I: Why did you use multiplication instead of division?
R: Ah… Is that the rule? I think it states that in dividing fractions, you only have to get the reciprocal of the second fraction and multiply them.
I: So, you will just ask your students to follow the algorithm and accept it as is?
R: No, but it is very confusing.

I: You got the correct answer, but what if students ask you why the product is smaller than the value of the factors? How will you explain that to your students?
R: I could use overlapping squares. I think it is done by showing the grids, using acetates. We could count how many squares are occupied. With this picture, we can also show that 12/24 is equal to ½. (But was not able to show it)

Students tend to forget the algorithm of multiplication and division of fractions because they were simply asked to memorize the procedure which had not been fully explained or justified to them. Therefore, at a later time after they have learned the algorithm, they still tend to follow the same procedure as in whole numbers, they multiply or divide numerator with numerator and denominator with denominator.

On Arithmetic/Geometric Progression, the MDAS Rule and on showing why –(-x) = x. Six (54%) out of eleven respondents were not able to solve the arithmetic and geometric progressions. They could not even recall the formulas. The same is also true with the MDAS Rule and on showing why the negative of a negative number is its positive. Most of the student respondents were unanimous in their expression that they were simply following the rule or the formula and in their agreement that the said concepts are based on some logical proof. However, not one of them was able to give strong arguments as to how the formula or proof was derived. Here are some transcriptions of their responses.

I: How will you explain the process that you went through in solving these arithmetic and geometric progressions?
R: I don't know. I just followed the formula.
I : How were you able to solve the numerical expression?
R: I followed the MDAS rule.
I : Why?
R: From our previous experience. We were taught what the early mathematicians did.
I : But have you ever wondered if the mathematicians really made the right choice in adapting the MDAS rule as the correct one?
R: I'm already asking myself why it is wrong. I'm confused, so we're just following the MDAS rule.

I: How will you explain to your students that –(-x) = x?
R: It is a rule. My teacher said so.

On division of Zero. Six (54%) out of eleven respondents have an idea of what the concept of infinity is and of the quotient being undefined but none were able to express themselves in a convincing manner. One seemed to be on the verge of making a discovery in the discussion of his knowledge of these concepts.

I: You answered 0/2 = 0. Will the quotient be always equal to zero?
R: If we simply divide a pie into 2, but you wont take any of these. So, the answer is zero.
I: You said 2/0 is undefined. How will you explain what is undefined to your students?
R: If we have a pie again but we don't know what part of the pie is taken. So the answer is undefined.

On the Zero Power of any number or variable. Eight (73%) out of eleven respondents were able to offer strong arguments in support of the concept whatever the given number or variable. They were able to make use of their prior knowledge on the expanded form of a number in exponential form and the law of exponents regarding division in their explanations and apply the concept to other possibilities such as substituting the number 2 in the expression.
I: How are you going to explain this to your students?

R: Using the example $\frac{2^3}{2^3}$, we know that $2^3 = 2x2x2$, so $\frac{2x2x2}{2x2x2}$ is already equal to 1. In the division of expressions in exponential form, the rule is $\frac{x^m}{x^n} = x^{m-n}$. Hence, applying that to $\frac{2^3}{2^3}$, then $\frac{2^3}{2^3} = 2^{3-3} = 2^0 = 1$.

These are quite alarming results. Mathematics teachers, to be effective, do not only need to have a repertoire of algorithmic knowledge. Knowledge of procedures is necessary but this is not sufficient. An effective teacher must also have the ability to translate information into meaning, which learners will understand, or what is known as their conceptual pedagogy. Their mathematical understanding does not consist only of procedural facility, but also an understanding of how and why the algorithm works, and what they are able to connect in their students' schema and make use of in new situations in the course of teaching (Ball, 2000; Bromme & Steinbring, 1994; Bryan, n.d.; Clarke & Biddle, 1993; Entwistle, 1998; Fairbanks, Freedman, & Kahn, 2000; Long & Temple, 1996; Kauchak & Eggen, 1998; Kinach, 2002; Yumus, 2001).

Conclusions

Many of the mathematical tasks explored during the interview are rather elementary in nature but proved to be deceptively difficult to consider from a conceptual perspective. The level of reasoning demonstrated by pre-service school teacher respondents was generally low. It seems that that the four years spent in their study program is insufficient for them to gain insights into the concepts they have encountered in school mathematics. Several factors could have contributed to this problem. First, teachers hesitate to spend much time on making mathematics meaningful for students due to content and time constraints. The pressure of achieving good results in national exams results in teachers giving more drills to students on mathematical concepts rather than helping them to understand the concept. Second, the mathematics taught in the university prepares the students to be involved in the discipline of mathematics as a profession or for continuing education and not much emphasis is given to strengthening their mastery of school related concepts which they are going to impart to their future students.

Reasoning ability can not be developed in a specific math course (Prevost, 2000). It has to be developed throughout the entire education program curriculum. Educators know that changes in student outcomes are the result of changes in curriculum and instruction. However, it is apparent that many teachers today are caught in the midst of change for which they may not have been professionally prepared. Many were educated in classrooms where the role of the student was to memorize information, conduct well-regulated experiments, perform mathematical calculations using a specific algorithm, and were then tested on their ability to repeat or remember specific facts. As Fuhrman said "Students deserve teachers who have depth of conceptual understanding of specific content areas, which they are able, in turn, to share with students in the classes they teach, and TIMSS-R reminds us that this is an area in which progress is still to be made". (TIMSS-R, 2000)

Critical analysis of the quality of mathematical reasoning of pre-service school teachers should come up with specific measures to improve the performance of students in mathematics. In making thinking and reasoning the cornerstone of mathematics instruction, the total commitment of the teacher is needed, that is, they should be able to *REASON: R*edirect students' observations and prior knowledge to critical variables that give depth to learning, *E*nhance critical thinking skills through the teacher's art of questioning, *A*pply knowledge acquired to real life situations, *S*trategize students' learning experiences, *O*ptimize students' participation in the learning process and *N*urture students' understanding of concepts for meaningful learning without resorting to rote memorization.

Mathematical reasoning needs to be emphasized in the classroom. Students should be encouraged to ask questions and to argue when something is not clear to them. This is the opposite of the students' usual acceptance that the "teacher is always right", "the book is never wrong" and "mathematics is like that" (Yumus, 2001). Teachers should expect their students to explain and justify their answers, assist them in considering and evaluating several solutions to a problem and create a classroom environment where students feel comfortable questioning, challenging, suspending judgment and demanding reasons and justifications as they deal with mathematical and real-world content. On the other hand, teachers must be well-equipped with conceptual pedagogy, to be able to translate instrumental understanding of their students to relational understanding (Ball, 2000; Bromme & Steinbring, 1994; Bryan, n.d.; Doerr, 2003; Kinach, 2002; Yumus. 2001). They can only do this by revisiting their own mathematical reasoning- their understanding of school mathematics. These are the measures that can better upgrade the teaching of mathematical tasks, thus facilitating students' learning and understanding.

In line with the foregoing, mathematics education programs must include such courses as "Insights of School Mathematics Interrelatedness and its Relation to other Sciences" and "Analysis and Evaluation of Teaching and Learning School Mathematics", through which teachers could assess the effectiveness of the pedagogical content knowledge they are adapting. In their training programs, they should learn how to plan their strategy for solving a problem, learn how to break up long trains of reasoning into manageable steps, learn

how to find counter-examples for false mathematical statements, learn how to discern between false and true mathematical statements and be able to communicate clearly mathematical reasoning, both orally and in writing (URL 1). Mathematics will continue on its present course if pre-service teachers do not acquire and learn how to apply reasoning skills well and curriculum planners do not respond to the need of developing mathematical reasoning in teacher development programs.

References

Ball, D. L. (2000). Bridging practices: intertwining content and pedagogy in teaching and learning to teach. *The Journal of Policy, Practice and Research in Teacher Education, 51*(3), 241-247.

Baroody, A. J. (1993). *Problem solving, reasoning and communicating Helping children think mathematically*. New York: Mcmillan Publishing Co.

Bigge, M., & Shermis, S. (1999). *Learning theories for teachers*. New York: Addison Wesley Longman Inc

Bromme, R., & Steinbring, H. (1994). Interactive development of subject matter in the mathematics classroom. *Educational Studies in Mathematics, 27*(3), 10-13, 144-150, 212-217.

Bryan, T. J. (n.d.). The conceptual knowledge of preservice secondary mathematics teachers: how well do they know the subject matter they will teach? *Issues in the Undergraduate Mathematics Preparation of school Teachers.* Retrieved Feb. 12, 2002, from http:www. k-12prep.math.ttu.edu/journal/contentknowledge/bryan01/article.pdf

Clarke, J. H., & Biddle, A. W. (1993). *Teaching critical thinking reports from across the curriculum.* Englewood Cliffs, New Jersey: Prentice Hall Studies.

Cooney, T. J. (1998). Conceptualizing belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics Education, 29*(3), 306-333.

de Castro, B. (2003). Teachers' roadmap to students' better understanding through mathematical reasoning. *Unitas. 75*(2), 287-300.

Doerr, H. (2003). A modeling perspective on students' mathematical reasoning about data. *Journal for Research on Mathematics Education, 32* (2), 110–136.

Education Issues Series. (1996). Quezon City, Philippines: UP National Institute for Science and Mathematics Education Development.

Entwistle, N. (1998, Nov. 5). *Conceptions of learning, understanding and teaching in higher education.* Retrieved May 22, 2002, from http://www.scre.ac.uk/fellow/fellow98/entwistle.html .

Ernest, P. (Ed.). (1994). *Constructing math knowledge: epistemology and mathematical education.* Falmer Press.

Fairbanks, C., Freedman, D. & Kahn, C. (2000). The role of effective mentors in learning to teach. *Journal of Teacher Education, 51*(2), 102-111.

Fraivillig, J. L. (1999). Advancing children's mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematiucs Education, 30*(2), 148-170.

Greenes, C. (1995). Mathematics learning and knowing: a cognitive process. *Journal of Education*, *177*(1), 85-105.

Hemingsen, M. & Stein, M. (1997). Mathematical tasks and student cognition: classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524-549.

Ibe, M. D. (2001). Confluence of mathematics teaching methods, textbook preparation and testing: explaining RP TIMSS student performance. *Intersection Proceedings Biennial Conference. Curricular Issues and New Technologies.* Phil. Council of Mathematics Teachers Educators MATHTED Inc.

Jones, E. D., Wilson, R., & Bhojwani, S. (1997). Mathematics instruction for secondary students with learning disabilities [Electronic version]. *Journal of Learning Disabilities*, *30*(2). Retrieved May 21, 2002, from http://www.Idoline.org/ld.indepth/mathskills/math/mathild.html .

Kauchak, D. P., & Eggen, Paul D. (1998). *Learning and teaching research-based methods* 3rd ed. Needham Heights, Massachusetts: Allyn & Bacon.

Kinach, B. M. (2002). A cognitive strategy for developing pedagogical content knowledge in the secondary mathematics methods course: toward a model of effective practice. *Teaching and Teacher Education, 18*(1), 51-71. Retrieved December 4, 2002 from http://www.elsevier.com/locate/tate.

Krulik, S., & Rudnick, J. (1996). *The new sourcebook for teaching reasoning and problem solving in junior and senior high school*. Needham Heights, Massachusetts: Allyn & Bacon.

Long, C., & Temple, D. (1996). *Mathematical reasoning for elementary teachers.* New York: Harper Collins Publisher.

Murray, H. (n. d.). *In-depth assessment of children's mathematical reasoning*. Retrieved May 10, 2002 from http//:www.nku.edu/~sheffield/murray.html

Mewborn, D. S. (1999). Reflective thinking among preservice elementary mathematics teachers. *Journal for Research in Mathematics Education*, *30*(3), 316-341.

NCTM standards (2000). Retrieved May 10, 2002 from http://www.standards.nctm.org/document.

Orton, R. E. (1997). Toward an aristotelian model of teacher reasoning. *Curriculum Studies*, *29* (5), 569 -583.

Prevost, F. J. (2000). *Rethinking how we teach: learning mathematical pedagogy*. Retrieved May 10, 2002, from http://www.enc.org/online/ENC2174/2174.html

Raimi, R. (2002, Oct.). *On mathematical reasoning in school mathematics*. A Mathematical Manifesto. New York City Hold News Page. Retrieved May 16, 2003, from http//:www.nychold.com/raimi-reason02.html

Siegel, H. (1988). *Philosophy of education: educating reason.* New York: Routledge Inc.

Steen, L. A. (1999). *Twenty questions about mathematical reasoning.* Retrieved June 24, 2002 from http://www.stolf.edu/people/steen/papers/reason.html .

TIMSS-R results. (2000). Retrieved April 15, 2002 from http://nces.ed.gov/timss/results.asp.

Vacc, N., & Bright, G. (1999). Elementary preservice teachers' changing beliefs and instructional use of children's mathematical thinking. .*Journal for Research in Mathematics Education*, *30*(1), 89-110.

Yumus, Dr. A. S. (2001). Levels of reasoning in mathematical concepts among preservice mathematics teachers. 2nd APED International Conference in Education Research Understanding Eduactional Issues in the Asia Pacific Region Universals, Uniqueness and Cooperation. Seoul, Korea, 52-61.

Received Mach 29, 2004
Revision received November 11, 2004
Accepted December 20, 2004